

Product Quality Statistical Control in a Tool-Wear Process

Controllo Statistico della Qualità di Prodotto Uscente da un Processo Produttivo Soggetto all'Usura dei Macchinari

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Riassunto: Il comportamento di molti processi produttivi rappresentato mediante carte di controllo evidenzia un marcato fenomeno di deriva nel valore medio della caratteristica sotto indagine: sono processi con macchinari soggetti ad usura. Le carte di controllo utilizzate per tali processi hanno limiti di controllo allargati, in quanto alla variabilità naturale si aggiunge la variabilità dovuta alla deriva sistematica. La letteratura propone differenti procedure per compensare tali comportamenti sistematici. In questo lavoro, si propone un approccio differente: in fase di definizione della capacità del processo si avvia uno studio della deriva attraverso modelli tipici della regressione segmentata e poi, tenendo conto della forma della deriva stimata, il manufatto viene classificato con diversi livelli di qualità, dipendenti dal livello di usura del macchinario.

Keywords: Tool-Wear Process, Statistical Process Control, Segmented Regression

1. Introduction and background

There are a number of processes for which the conventional procedures of checking their statistical control are not quoted as the standard ones. We are referring to a metal machining operation that mould production parts using, for example, a cutting operation such as a metal lathe, a drill bit, and so on. The used tool is sensitive to wear and tear and some adjustment are usually made during the tool life. Therefore the pattern in a control chart, for example in a Shewhart control chart, is not so random, but it exhibits a special path. The simplest pattern observed in most of such a processes is a linear trend between the characteristic measurements and the order of the production (or the sampling order). The linear trend was acknowledged already in the past (Manuele, 1945; Mandel, 1969; Quesenberry, 1988) and many books (Duncan, 1974; Vaughn, 1974; Grant and Leavenworth, 1980; Montgomery, 2005) deal with processes with such a feature. Also the standards are aware of the existence and of the impossibility to leave out of consideration special but actual experimental pattern in the control charts. The German Standards DIN 55 319 (2002) defines eight different typologies of a process. The named C Model deals with processes whose mean is time dependent; particularly, the C3 Model (as an example, see Figure 1 regarding the case studied) deals with a linear pattern of the process mean, being normal the instantaneous distribution of the characteristics.

The aim of most the literature is addressed to locate the most convenient moment of resetting the process because resetting the process is expensive and time consuming. As a consequence, the attempt is in minimizing the number of the resettings required for the process to produce all, or most, of the parts within specification limits. If one wants

to use these techniques efficaciously, the specification limits have to be enlarged of a number of multiple of the process standard deviation σ about the linear trend line. Manuele (1945) suggested specification limits are approximately 13σ , Duncan (1986) about 13σ ; in any case no authors suggest limits less than 9σ or 10σ .

In this paper the aim is addressed to estimate the actual trend and to take action according to it. In the next section, this different approach is proposed both for defining the process capability and for checking different quality levels of the manufactured parts. In the last section the result of a case study are presented.

2. Modelling a Tool-Wear Process

Let denote the observed values of the characteristic of interest in n consecutive samples of equal size k by:

$$Y_{11}, Y_{12}, \dots, Y_{1k}, Y_{21}, \dots, Y_{2k}, \dots, Y_{n1}, \dots, Y_{nk} \quad (1)$$

If the process is in control, the usual model for the mean μ of the process is:

$$Y_{ij} = \mu + \varepsilon_{ij} \quad (2)$$

with $i = 1, 2, \dots, n$; $j = 1, 2, \dots, k$. The error components ε_{ij} are supposed to be independently, identically normal distributed with zero mean and constant variance σ^2 . Assuming that over at least part of the production range the life of a tool can be modeled by a simple linear regression function, the suggested model is a segmented linear model that can be written as:

$$\begin{cases} Y_{ij} = \mu + \varepsilon_{ij}, & \forall i \leq i_1 \wedge j \leq j_1 \\ Y_{ij} = \mu + [(i - i_1)k + (j - j_1)]\beta_1 + \varepsilon_{ij}, & \forall i_1 < i \leq i_2 \text{ and } j_1 < j \leq j_2 \\ Y_{ij} = \mu + [(i_2 - i_1)k + (j_2 - j_1)]\beta_1 + [(i - i_2)k + (j - j_2)]\beta_2 + \varepsilon_{ij}, & \forall i > i_2 \text{ and } j > j_2 \\ \dots\dots \end{cases} \quad (3)$$

The rationale for this model is that the observed period of the process can be divided in sub periods, according to the plot of a run chart. Initially, the effects of the tool-wear of the machinery are nearly inexistent, so the actual observed values of the sample means are realizations of the process mean apart from a white noise. When the tool-wear becomes evident, the trend of the observed sample means can be modeled by a line. Unfortunately, the linear trend doesn't show a constant slope. In the most common cases, the worse is the tool-wear the larger is the value of the slope. The requirement of the continuity among the different segmented lines seems to be reasonable and it does not lack of generality, even if the number of parameters to be estimate increases over the first. In fact, the estimation problem involves not only the individual parameters of the segmented lines, but also the points (*break-points*) in which the trend changes the slope (in (3), they are expressed by i_l and j_m , with $l = 1, 2, \dots$ and $m = 1, 2, \dots$).

Moreover, the conventional $\bar{x}$ -chart may be replaced by an adequate number of $\bar{x}$ -charts, with different control limit lines because both of their values and of the non parallelism to the horizontal axis (apart from the first one). The central lines of such

charts corresponds with the estimates of the process averages; the upper and lower control limits are parallel to the central lines according to a standard deviation based on the deviations of the observed values about the individual segmented regression lines.

Therefore in accordance with (3), the estimates of the process averages, as many as the number of the segmented line, are respectively: $\hat{\mu}_1 = \frac{1}{kn_1} \sum_{i=1}^{n_1} \sum_{j=1}^k y_{ij}$,

$\hat{\mu}_2 = \hat{\mu}_1 + [n_1 k + (k+1)/2] \hat{\beta}_1$, $\hat{\mu}_3 = \hat{\mu}_1 + n_2 k \hat{\beta}_1 + [k(n_3+1)/2 - (k-1)/2] \hat{\beta}_2, \dots$ (for the sake of simplicity, it can be assumed from now on that $\hat{j}^* = \hat{j}^{**} = k$, $\hat{i}^* = n_1$, $\hat{i}^{**} = n_1 + n_2, \dots$, with $n = \sum_l n_l$; it is equivalent to approximate the estimated break-

points with the last elements of the sample respectively $n_1, n_2, \dots$). An estimate of σ^2 (variance of the process in control) can be obtained only from the observed values when the tool-wear was inexistent, pooling the estimated within samples variances:

$\hat{\sigma}^2 = \frac{1}{n_1(k-1)} \sum_{i=1}^k \sum_{j=1}^{n_1} (y_{ij} - \bar{y}_{i\bullet})^2$. When the tool-wear enlarges, the variance of the

process is inflating and, according the (3), is for the l -th segmented regression line:

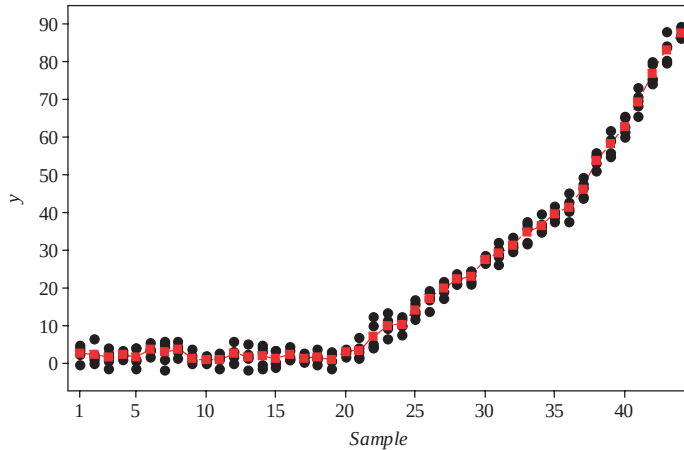
$\hat{\sigma}^2 + \frac{1}{k-1} \hat{\beta}_l^2 \sum_{j=1}^k (j - \bar{j})^2$, with $l = 1, 2, \dots$

Correspondingly, the quality of the manufactured parts may be classified at different levels according to the different $\bar{x}$ -charts: the highest level correspond to the first $\bar{x}$ -chart and then the quality, measured in terms of target values and specification limits, drops off.

3. A Case Study

The described procedure has been applied to the data collected in a firm in the automotive field. The machinery operation shaping metal sheets uses a mould, subject to wear. Therefore, the dimensions of the parts change in a predictable pattern (Fig. 1).

Figure 1: The run chart of the observed values in the shaping metal sheet machinery exhibits the behavior of a linear trend process.



In Table 1 there the estimates obtained using the Segmented Package of Software R. Leaving out of consideration the summary of the residuals confirming reasonably their normal distribution, the software estimated three segmented lines (few shots were necessary). There are the estimates of the process mean $\hat{\mu}$, of the two slopes $\hat{\beta}_1$ and $\hat{\beta}_2$ that can be assumed equal respectively 2, 0.5 and 1.2 (the corresponding tests cannot be rejected). The software also estimates the slope for the first segmented line ($\hat{\beta}_0 = 0.001511$), but according to the p -value it can be assumed equal zero. The estimates of the two breaking points (98.1 and 181.45) can be approximated with 100 and 180 corresponding respectively to the twentieth sample and to the thirty-sixth one having each sample a size $k = 5$. Also an estimate of the variance of the process σ^2 is supplied ($\hat{\sigma} = 2.008$ on 214 degrees of freedom); the inflated variances may be computed according the formulas in section 2.

Table 1: Output of the Software R Segmented Package for the observed value in Figure 1.

Residuals:				
Min	1 st Q	Median	3 rd Q	Max
−5.28053	−1.33517	0.04885	1.39301	5.38063
Coefficient:				
	Estimate	Std. Error	t value	Pr(> t)
Intercept ($\hat{\mu}$)	1.963	0.4066	4.28	2.62e-06
Slope 0 ($\hat{\beta}_0$)	0.001511	0.00706	0.214	0.83
Slope 1 ($\hat{\beta}_1$)	0.4976	0.01216	40.909	<2e-06
Slope 2 ($\hat{\beta}_2 - \hat{\beta}_1$)	0.7077	0.02741	25.822	<2e-06
Psi1	98.1	1.218	80.54	<2e-06
Psi2	181.45	1.076	168.63	<2e-06
Residual standard error ($\hat{\sigma}$): 2.008 on 214 degrees of freedom				
Multiple R-Squared: 0.994			Adjusted R-Squared: 0.9938	
F-statistic: 7061.0n 5 and 214 DF, p-value: <2e-06				

References

- DIN 55 319 (2002), *Qualität sfähigkeits kenn Größen*, März 2002, Deutsches Institut für Normung, Deutschland.
- Duncan, A. J. (1986), *Quality Control and Industrial Statistics*, 5th ed. Richard D. Irwin, Homewood, Ill.
- Grant, E. L. and Leavenworth, R. S. (1980), *Statistical Quality Control*, 5th ed. McGraw-Hill, New York, NY.
- Mandel, J. (1969), The Regression Control Chart, *Journal of Quality Technology*, 1, 1-9.
- Manuele, J. (1945), Control Chart for Determining Tool Wear, *Industrial Quality Control*, 1, 7-10.
- Montgomery, D.C. (2005), *Controllo Statistico della Qualità*, 5th ed. McGraw-Hill, MI, IT.
- Quesenberry, C.P. (1988), An SPC Approach to Compensating a Tool-wear Process, *Journal of Quality Technology*, 20, 220-229.
- Vaughn, R. C. (1974), *Quality Control*, 1st ed. Iowa State University Press, Ames, IA.